On the Lorentz Variance of the Claimed O(3)-Symmetry Law

— a Remark on a Former Article [2] in this Journal —

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Abstract. In 1992 M.W. Evans proposed a so-called $O(3)$ symmetry of electromagnetic fields by adding a constant longitudinal ghost field to the well-known transversal plane em waves. Evans considered this symmetry as a *new law of electromagnetics.* In 2000 he tried to show the Lorentz invariance of $O(3)$ symmetry of electromagnetic fields in this Journal [2]. However, this proof lacks from a simple calculation error. As shall be shown below a correct calculation yields no Lorentz invariance.
This result is of importance as later on, since 2002, this $O(3)$ symmetry became the center of M.W. Evans' CGUFT which he recently renamed as ECE Theory. – A law of Physics must be invariant under admissible coordinate transforms, namely under Lorentz transforms. Therefore, to check the validity of M.W. Evans' $O(3)$ symmetry law, we apply a longitudinal Lorentz transform to M.W. Evans' plane em wave (the ghost field included). As is well-known from SRT and recalled here the transversal amplitude decreases while the additional longitudinal field remains unchanged. Thus, M.W. Evans' $O(3)$ symmetry cannot be invariant under (longitudinal) Lorentz transforms: M.W. Evans**'** $O(3)$ **symmetry is no valid law of Physics.**

A law of Physics must be invariant under admissible coordinate transforms, namely under Lorentz transforms. A plane wave remains a plane wave also when seen from arbitrary other inertial systems. Therefore, Evans' $O(3)$ symmetry law should be valid in all inertial systems. Therefore to check the validity of Evans' $O(3)$ symmetry law in other inertial systems, we apply a longitudinal Lorentz transform to Evans' plane em wave (the ghost field included). As is well–known from SRT and recalled here the transversal amplitude decreases while the additional longitudinal field remains unchanged. Thus, Evans' $O(3)$ symmetry cannot be invariant under (longitudinal) Lorentz transforms: Evans' $O(3)$ **symmetry is not a valid law of Physics.**

In the following text quotations from M.W. Evans' GCUFT book [1] appear with equation labels [1.nn] at the left margin.

1. M.W. Evans' $O(3)$ hypothesis

The assertion of $O(3)$ symmetry is a central concern of M.W. Evans' considerations since 1992: He claims that each plane circularly polarized electromagnetic wave $\mathbf{B}$ is accompanied by a constant longitudinal field $\mathbf{B}^{(3)}$, a so-called "ghost field".

Evans considers a circularly polarized plane electromagnetic wave propagating in z-direction, cf. [1; Chap.1.2] . Using the electromagnetic phase

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[1.38] $$\Phi = \omega t - \kappa z,$$

where $\kappa = \omega/c$, M.W. EVANS describes the wave relative to his complex circular basis [1;(1.41)]. The magnetic field is given by

[1.43/1] $$\mathbf{B}^{(1)} = \tfrac{1}{\sqrt{2}} B^{(0)} (\mathbf{i} - i\mathbf{j}) e^{i\Phi},$$

[1.43/2] $$\mathbf{B}^{(2)} = \tfrac{1}{\sqrt{2}} B^{(0)} (\mathbf{i} + i\mathbf{j}) e^{-i\Phi},$$

[1.43/3] $$\mathbf{B}^{(3)} = B^{(0)} \mathbf{k},$$

satisfying M.W. EVANS' "cyclic $O(3)$ symmetry relations"

[1.44/1] $$\mathbf{B}^{(1)} \times \mathbf{B}^{(2)} = i B^{(0)} \mathbf{B}^{(3)*},$$

[1.44/2] $$\mathbf{B}^{(2)} \times \mathbf{B}^{(3)} = i B^{(0)} \mathbf{B}^{(1)*},$$

[1.44/3] $$\mathbf{B}^{(3)} \times \mathbf{B}^{(1)} = i B^{(0)} \mathbf{B}^{(2)*}.$$

Especially equ.[1.43/3] defines M.W. EVANS' ghost field $\mathbf{B}^{(3)}$, which is coupled by the relations [1.44] with the transversal $\mathbf{B}$ components given by the two eqns.[1.43/1-2]. Evans' **B Cyclic Theorem** is the statement that each plane circularly polarized wave [1.43/1-2] is accompanied by a longitudinal field [1.43/3], and the associated fields fulfil the Cyclic equations [1.44]. Evans considers this $O(3)$ hypothesis¡/b¿ as a **Law of Physics**.

2. Checking the Lorentz invariance of the $O(3)$ hypothesis

In the article [2; p.14] M.W. EVANS therefore tries to prove the Lorentz invariance of the $O(3)$ hypothesis [1.44] by referring to the invariance of the vector potential $\mathbf{A}$ under Lorentz transforms. That is a good method obtaining the transform of ED fields if one calculates *correctly*.

So the reader should first check that the vector potentials of the transversal components $\mathbf{B}^{(1)}$ and $\mathbf{B}^{(2)}$ of the plane wave under consideration are given by

(2.1) $$\mathbf{A}^{(1)} = \frac{1}{\kappa}\mathbf{B}^{(1)} = \frac{1}{\kappa\sqrt{2}} B^{(0)} (\mathbf{i} - i\mathbf{j}) e^{i\Phi}, \ \mathbf{A}^{(2)} = \frac{1}{\kappa}\mathbf{B}^{(2)} = \frac{1}{\kappa\sqrt{2}} B^{(0)} (\mathbf{i} + i\mathbf{j}) e^{-i\Phi}$$

while the vector potential of the *longitudinal* field $\mathbf{B}^{(3)}$ is

(2.2) $$\mathbf{A}^{(3)} = \frac{1}{2}\mathbf{B}^{(3)} \times (x\mathbf{i} + y\mathbf{j}).$$

The invariance of the vector potential $\mathbf{A}^{(3)}$ yields the invariance of $\mathbf{B}^{(3)}$ as well for *longitudinal* Lorentz transforms between inertial frames K, K' where K' moves relative to K with velocity $\mathbf{v} \parallel \mathbf{k}$ and $\beta := |\mathbf{v}|/c$.

What M.W. EVANS has obviously ignored in [2]: Frequency ω and wave number κ are *no invariants*. Under *longitudinal* Lorentz transforms we have the well-known Doppler effect:

(2.3) $$\omega' = \sqrt{\frac{1-\beta}{1+\beta}}\,\omega, \qquad \kappa' = \sqrt{\frac{1-\beta}{1+\beta}}\,\kappa.$$

Therefore the invariance of the vector potentials doesn't transfer to the transversal field components $\mathbf{B}^{(1)}$ and $\mathbf{B}^{(2)}$.

Due to the invariance of $\mathbf{A}$ we obtain from (2.1)

$$\text{(2.4)}\quad \mathbf{B}'^{(1)} \times \mathbf{B}'^{(2)} = \kappa'^2 \mathbf{A}'^{(1)} \times \mathbf{A}'^{(2)} = \frac{\kappa'^2}{\kappa^2} \kappa^2 \mathbf{A}^{(1)} \times \mathbf{A}^{(2)} = \frac{1-\beta}{1+\beta}\, \mathbf{B}^{(1)} \times \mathbf{B}^{(2)},$$

i.e. the expression $\mathbf{B}^{(1)} \times \mathbf{B}^{(2)}$ at the lefthand side of [1.44/1] does not remain invariant while $\mathbf{B}^{(3)}$ is invariant due to (2.2). Hence equ.[1.44/1], if valid in the inertial system K, *cannot be valid* also in the inertial system K': Evans' cyclical symmetry [1.44] is *not Lorentz invariant and cannot be a Law of Physics.*

References

[1] M.W. Evans, Generally Covariant Unified Field Theory, the geometrization of physics; Arima 2006

[2] M.W. Evans, On the Application of the Lorentz Transformation in $O(3)$ Electrodynamics, APEIRON Vol.7 2000, 14-16
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[3] G.W. Bruhn and A. Lakhtakia, Commentary on Myron W. Evans' paper "The Electromagnetic Sector ...",
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